$$f(x) = \frac{|x-2|}{x}$$

Using algebra in this context means to turn the function into a piecewise function + then analyze that function.

$f(x)$ does one thing due to the absolute value when $x-2 \geq 0$, and another when $x-2<0$.

Therefore,

$$f(x) = \begin{cases} \frac{x-2}{x} & \text{if } x-2 \geq 0 \\ \frac{-(x-2)}{x} & \text{if } x-2<0. \end{cases}$$

In proper form,

$$f(x) = \begin{cases} \frac{x-2}{x} & \text{if } x \geq 2 \\ \frac{-(x-2)}{x} & \text{if } x<2. \end{cases}$$

Then the right + left sided limits simply follow.

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} \frac{x-2}{x} = \frac{\lim_{x\to 2} x - \lim_{x\to 2} 2}{\lim_{x\to 2} x} = \frac{2-2}{2} = 0$$

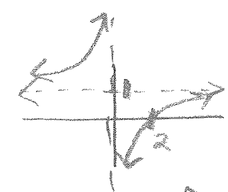
$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} \frac{-(x-2)}{x} = \frac{\lim_{x\to 2} -x + \lim_{x\to 2} 2}{\lim_{x\to 2} x} = \frac{-2+2}{2} = 0$$

Since $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$,

$\lim_{x \to 2} f(x)$ exists and equals 0.

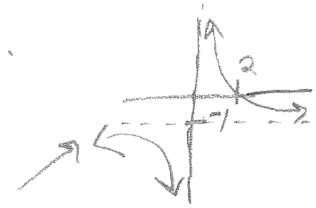
To support with a sketch, we need to graph the piecewise function $f(x)$.

For $x \geq 2$, we graph $\frac{x-2}{x} = 1 - \frac{2}{x}$.



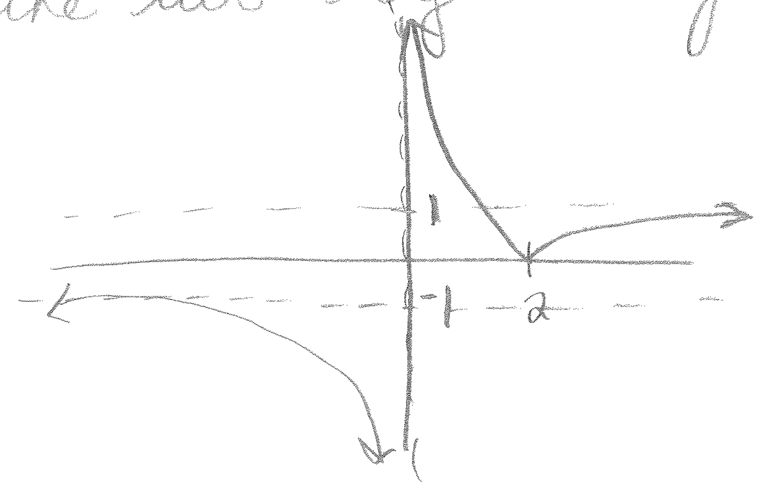
That is, $\frac{2}{x}$ flipped vertically + shifted up 1.

For $x < 2$, we graph $\frac{-x+2}{x} = -1 + \frac{2}{x}$.



That is, $\frac{2}{x}$ shifted down one unit.

Putting the two together yields:



You can see, at $x = 2$, the limits from both sides are 0.